QUESTION BANK

BA PROGRAMME , CALCULUS

Q 1 Discuss the existence of the limit of the function :

$$f(x) = \begin{cases} x & x \leq 1 \\ 2 - x & 1 < x \leq 2 \\ -2 + 3x - x^2 & x > 2 \end{cases}$$

at $x = 1$ and $x = 2$.

Q 2 Find a value for the constant k, if possible, that makes the function f continuous everywhere where f is defined by :

$$f(x) = \begin{cases} 7x - 2 & x \leq 1 \\ kx^2 & x > 1 \end{cases}$$

Q 3 Prove that the function f defined by

$$f(x) = \begin{cases} \dfrac{x}{e^{\frac{1}{x}} + 1} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is not differentiable at origin.

Q 4 Find the nth derivative of $\sin^2 x \cos^3 x$.

Q 5 If $y = \sin^{-1} x$, then

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0.$$

Q 6 If

$$z = \tan^{-1} \frac{x^3 + y^3}{x - y},$$

then using Euler's theorem, prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sin 2z.$$

Q 7 State Euler's theorem and verify Rolle's theorem for the function

$$f(x) = (x - a)^m (x - b)^n, \quad x \in [a, b]$$

where m and n are positive integers.

Q 8 Prove that :

$$\frac{\tan x}{x} > \frac{x}{\sin x}, \quad 0 < x < \frac{\pi}{2}.$$

Q 9 Obtain the Maclaurin's infinite series expansion of e^x, $x \in \mathbf{R}$.

Q 10 Find the maximum and minimum values of $(1 - x^2)e^x$.

Q 11 Determine the values of p and q for which :

$$\lim_{x\to 0} \frac{x(1 + p\cos x) - q\cos x}{x^3}$$

exists and is finite.

Q 12 Evaluate :

$$\lim_{x\to 0} (1 + x)^{\frac{1}{x}}.$$

Q 13 Find the equation of tangent to the parabola $y^2 = 4x + 5$, which is parallel to line $y - 2x + 3 = 0$.

Q 14 If ρ_1, ρ_2 be the radius of curvature at the extremities of any chord of cardiode $r = a(1 + \cos\theta)$ which passes through the pole, then show t $\rho_1^2 + \rho_2^2 = 16a^2/9$.

Q 15 Find all the asymptotes of the curve :

$$x^2y - xy^2 + xy + y^2 + x - y = 0.$$

Q 16 Determine the position and nature of the double points on the curve :

$$x^3 - y^2 \div 7x^2 + 4y + 15x - 13 = 0.$$

Q 17 Trace the curve :

$$y^2(a + x) = x^2(3a - x).$$

Q 18 Prove that :

$$\lim_{x \to 0} \frac{x - |x|}{x}$$

does not exist.

Q 19 Examine the continuity of the function at $x = 0$ and $x = 1$ for

$$f(x) = \begin{cases} -x^2 & x \le 0 \\ 5x - 4 & 0 < x \le 1 \\ x^2 - 3x & x > 1 \end{cases}$$

Also state the kind of discontinuity, if any.

Q 20 Find the nth derivative of $\dfrac{x^2 + 4x + 1}{x^3 + 2x^2 - x - 2}$.

Q 21 If $y = e^{m \cos^{-1} x}$, then

$$(1 - x^2) y_{n+2} - (2n + 1) x\, y_{n+1} - (n^2 + m^2)\, y_n = 0.$$

Q 22 State Euler's theorem. Also verify Euler's theorem for :

$$z = \tan^{-1}\left(\frac{y}{x}\right).$$

Q 23 Find the equations of the tangents and the normal at the point $\theta = \pi/2$ of curve :

$$x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta).$$

Q 24 Prove that the sum of the intercept on the coordinate axis of the tangent to the cu $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant.

Q 25 Show that the curvature of a circle of radius a is $1/a$.

Q 26 State Rolle's theorem. Give the geometrical interpretation of Rolle's theorem a verify Rolle's theorem for the function $f(x) = (x - 2)(x + 1),\ x \in [-1, 2]$.

Q 27 Show that the function :

$$f(x) = 3x^3 - 9x^2 + 9x + 7$$

is strictly increasing everywhere.

Q 28 Find the maximum value of $\frac{1}{x^x}$.

Q 29 Prove that :

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\dots\dots,$$

for $-1 < x < 0$.